End of loop for i = A

Loop 2 for i = B (**Compartment** B)

Step 1: Calculate f_i = (–18– (–16,5))/(–18,9 – (–16,5)) = 0,625. Verify that this is higher than 0 and lower than 1. Result is OK.

Step 2: Calculate T_j values:

T_A = 5,5 + 0,625 × (2,4 – 5,5) = 3,56 °C

T_B = –16,5 + 0,625 × (–18,9 – (–16,5)) = –18,0 °C

T_C = 1,3 + 0,625 × (–2,0 – 1,3) = –0,76 °C

T_D = –10,7 + 0,625 × (–13,9 – (–10,7)) = –12,7 °C

Step 3: T_A less than or equal to target of 4 °C? Result: true

T_B less than or equal to target of –18 °C? Result: true

T_C less than or equal to target of 0 °C? Result: true

T_D less than or equal to target of –12 °C? Result: true

All interpolated temperatures are below target so the interpolated **energy consumption** can be calculated: $E_{B\text{-}tar}$ = 822,1 + 0,625 × (935,6 – 822,1) = 893,0 Wh/d.

End of loop for i = B

Loop 3 for i = C (**compartment** C)

Step 1: Calculate f_i = (0,0 –1,3)/(–2,0 – 1,3) = 0,394. Verify that this is higher than 0 and lower than 1. Result is OK.

Step 2: Calculate T_j values:

T_A = 5,5 + 0,394 × (2,4 – 5,5) = 4,28 °C; loop can be stopped as > 4 °C: $E_{C\text{-}tar}$ = invalid.

End of loop for i = C

Loop 4 for i = D (**compartment** D)

Step 1: Calculate f_i = (–12,0 –(–10,7))/(–13,9 – (–10,7)) = 0,406. Verify that this is higher than 0 and lower than 1. Result is OK.

Step 2: Calculate T_j values:

T_A = 5,5 + 0,406 × (2,4 – 5,5) = 4,24 °C; loop can be stopped as >4 °C: $E_{D\text{-}tar}$ = invalid.

End of loop for i = D

The final interpolated **energy consumption** is E_{linear} = minimum value of $E_{A\text{-}tar}$ to $E_{D\text{-}tar}$. As only $E_{B\text{-}tar}$ has a valid value, this is by definition the E_{linear} value (893 Wh/d).

Interpolation is on **Compartment** B and the slope S_i is –47,29 Wh/d/K.

The calculations for this example are shown in Table I.6 and are illustrated in Figure I.7. Moving from coldest to warmest, **Compartment** B (with energy E_2) is the first to cross its **target temperature** (while all other **compartments** are less than **target temperature**). The data can also be laid out in a table, which is useful when calculating the results using a spreadsheet. Blue text is where **compartment** temperatures are at or below target, red text are temperatures above target. Only loop 2 (**Compartment** B at target) is valid (column 3, energy in green text) as all **compartments** are at or below **target temperature**.

Table I.6 – Example of linear interpolation, results for four compartments

Parameter	Interpolation **Compartment** A (loop 1)	Interpolation **Compartment** B (loop 2)	Interpolation **Compartment** C (loop 3)	Interpolation **Compartment** D (loop 4)
f_i	0,483 87	0,625	0,393 94	0,406 25
Compartment A °C	4,0	3,562 5	4,278 8	4,240 6
Compartment B °C	–17,661	–18,0	–17,445	–17,475
Compartment C °C	–0,296 77	–0,762 5	0,0	–0,0406 25
Compartment D °C	–12,248	–12,7	–11,961	–12,0
Energy Wh/d interpolated	877,02	893,04	866,81	868,21

NOTE

– Green shading indicates interpolation at the **compartment target temperature**.

– Red text indicates that the **compartment** temperature is above the **target temperature** (not valid).

– Blue text indicates that the **compartment** temperature is at or below **target temperature** (valid).

– Red text for energy indicates an invalid value as one or more **compartment** temperatures are above **target temperature** for that interpolation.

– Green text for energy indicates a valid value as all **compartment** temperatures are at or below target for that interpolation.

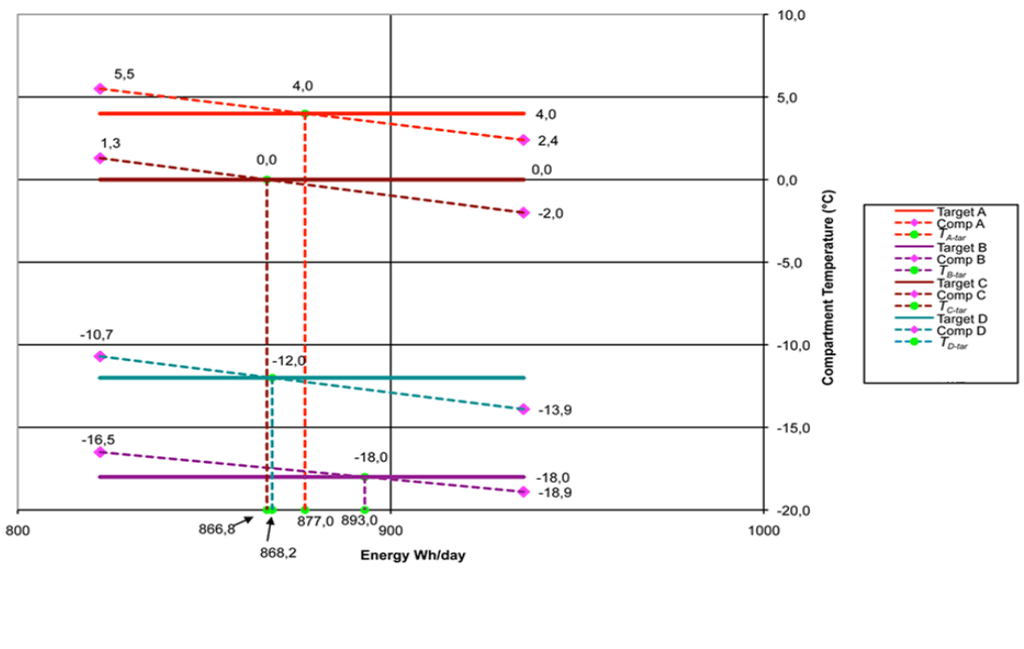

Figure I.7 – Example Interpolation for 4 compartments

I.3.3 Two compartments – manual triangulation

For this example, we consider a **refrigerator-freezer** with two **compartments** used for triangulation. The test data for 3 points is given in Table I.7. This example provides a worked example of the equations in E.4.

Table I.7 – Example of triangulation, two compartments

Parameter	Test 1	Test 2	Test 3	Point 4 (calc)	Type	Target
Compartment A	–20,7	–17,5	–16,0	–18,435 8	**Freezer**	–18,0
Compartment B	+6,5	+0,8	+7,1	+6,789	**Fresh Food**	+4,0
Energy Wh/d	1 390	1 310	1 120	1 259,93		

All 3 test points lie within the ±4 K of the **target temperature** for each **compartment**, so the points are valid. The 3 test points surround the intersection of the **target temperatures** (as illustrated in Figure I.8, so triangulation can proceed.

Firstly check that Point Q lies inside the triangle formed by test points 1, 2 and 3. Calculate the following two parameters as set out in E.4.2.2:

$$Check1 = [(T_{B-tar} - T_{B1}) \times (T_{A2} - T_{A1}) - (T_{A-tar} - T_{A1}) \times (T_{B2} - T_{B1})] \times [(T_{B-tar} - T_{B2}) \times (T_{A3} - T_{A2}) - (T_{A-tar} - T_{A2}) \times (T_{B3} - T_{B2})]$$

$$Check2 = [(T_{B-tar} - T_{B2}) \times (T_{A3} - T_{A2}) - (T_{A-tar} - T_{A2}) \times (T_{B3} - T_{B2})] \times [(T_{B-tar} - T_{B3}) \times (T_{A1} - T_{A3}) - (T_{A-tar} - T_{A3}) \times (T_{B1} - T_{B3})]$$

Point Q lies within the triangle formed by Points 1, 2 and 3 if the following inequality is true:

IF {[$Check1$ ≥ 0] AND [$Check2$ ≥ 0]} = TRUE (33)

NOTE It is recommended that these equations be entered into a spreadsheet for regular use to avoid errors. A value of 0 for Check1 or Check2 indicates that the Point Q lies exactly on one of the triangle sides and that linear interpolation could yield the same result with less data.

In this case, $Check1$ and $Check2$ yield the following results:

$$Check1 = [(4 - 6{,}5) \times (-17{,}5 - (-20{,}7)) - (-18 - (-20{,}7)) \times (0{,}8 - 6{,}5)] \times [(4 - 0{,}8) \times (-16 - (-17{,}5)) - (-18 - (-17{,}5)) \times (7{,}1 - 0{,}8)]$$

$Check1$ = 58,750 5

$$Check2 = [(4 - 0{,}8) \times (-16 - (-17{,}5)) - (-18 - (-17{,}5)) \times (7{,}1 - 0{,}8)] \times [(4 - 7{,}1) \times (-20{,}7 - (-16)) - (-18 - (-16)) \times (6{,}5 - 7{,}1)]$$

$Check2$ = 106,291 5

As both $Check1$ and $Check2$ are greater than 0, Point Q lies inside the triangle formed by Points 1, 2 and 3, so triangulation using manual interpolation or matrices can proceed.

An alternative approach to check that Point Q lies inside the triangle (using the same principles) is set out in E.4.6. Calculate the Determinant of each of the following four matrices:

D_0 for $\begin{vmatrix} -20{,}7 & 6{,}5 & 1 \\ -17{,}5 & 0{,}8 & 1 \\ -16{,}0 & 7{,}1 & 1 \end{vmatrix} = 28{,}71$

D_1 for
$$\begin{vmatrix} -18,0 & 4,0 & 1 \\ -17,5 & 0,8 & 1 \\ -16,0 & 7,1 & 1 \end{vmatrix} = 7,95$$

D_2 for
$$\begin{vmatrix} -20,7 & 6,5 & 1 \\ -18,0 & 4,0 & 1 \\ -16,0 & 7,1 & 1 \end{vmatrix} = 13,37$$

D_3 for
$$\begin{vmatrix} -20,7 & 6,5 & 1 \\ -17,5 & 0,8 & 1 \\ -18,0 & 4,0 & 1 \end{vmatrix} = 7,39$$

As a check $D_0 = D_1 + D_2 + D_3$

28,71 = 7,95 +13,37 + 7,39 = correct

If D_1 and D_2 and D_3 are the same sign as D_0, then Point Q is inside of the triangle (correct).

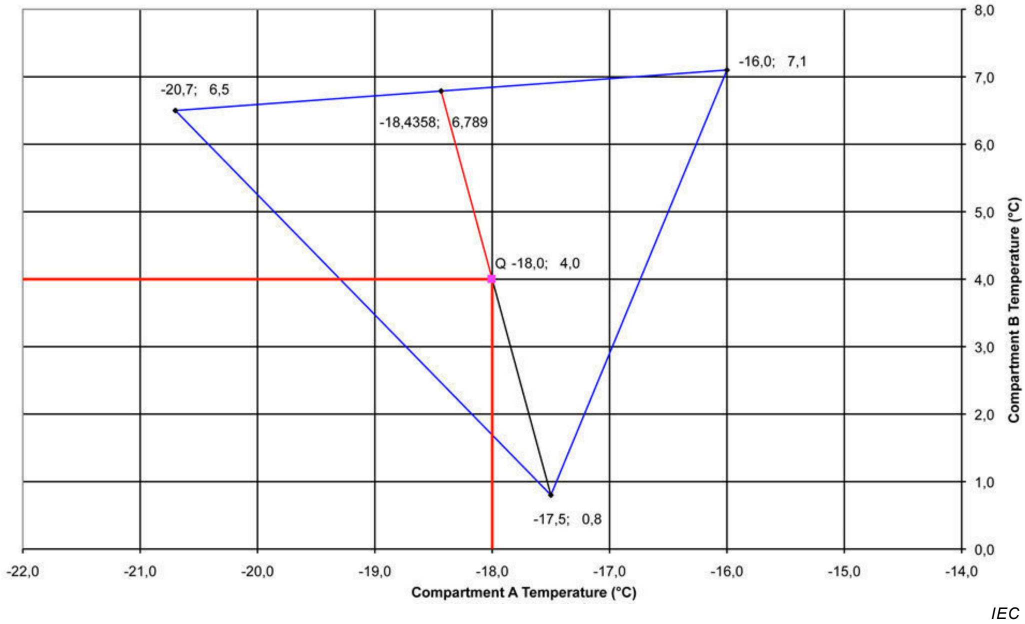

Figure I.8 – Example of triangulation (temperatures)

The equations to determine the values for manual interpolation are set out below.

Calculate the temperature in Compartment A at Point 4, which is the intersection of a line through Point 2 and Point Q (target) and a line between Points 1 and 3.

$$T_{A4} = \frac{\left[T_{B-tar} - \frac{T_{A-tar} \times (T_{B2} - T_{B-tar})}{(T_{A2} - T_{A-tar})} - T_{B1} + \frac{T_{A1} \times (T_{B3} - T_{B1})}{(T_{A3} - T_{A1})} \right]}{\left[\frac{(T_{B3} - T_{B1})}{(T_{A3} - T_{A1})} - \frac{(T_{B2} - T_{B-tar})}{(T_{A2} - T_{A-tar})} \right]} \quad (34)$$

$$T_{A4} = \frac{\left[4 - \frac{(-18{,}0)\times(0{,}8-4{,}0)}{((-17{,}5)-(-18{,}0))} - 6{,}5 + \frac{(-20{,}7)\times(7{,}1-6{,}5)}{((-16{,}0)-(-20{,}7))}\right]}{\left[\frac{(7{,}1-6{,}5)}{((-16{,}0)-(-20{,}7))} - \frac{(0{,}8-4{,}0)}{((-17{,}5)-(-18{,}0))}\right]} = -18{,}435\ 8\ °\text{C}$$

Figure I.8 shows clearly that Point Q lies within the triangle of test Points 1 to 3. Formula (33) above also confirms that Point Q lies inside the triangle formed by Points 1 to 3. An additional check may be performed as follows:

$T_{A4} < T_{A\text{-}tar} < T_{A2}$ or

$T_{A4} > T_{A\text{-}tar} > T_{A2}$

and

$T_{A1} < T_{A4} < T_{A3}$ or

$T_{A1} > T_{A4} > T_{A3}$

In this example the first condition of each is met:

–18,435 8 °C < –18 °C < –17,5 °C and

–20,7 °C < –18,435 8 °C < –16,0 °C

Where is there any doubt whether the Point Q lies inside the triangle (e.g. close to one of the sides of the triangle), mathematical evaluation in accordance with Formula (33) shall be used to confirm validity.

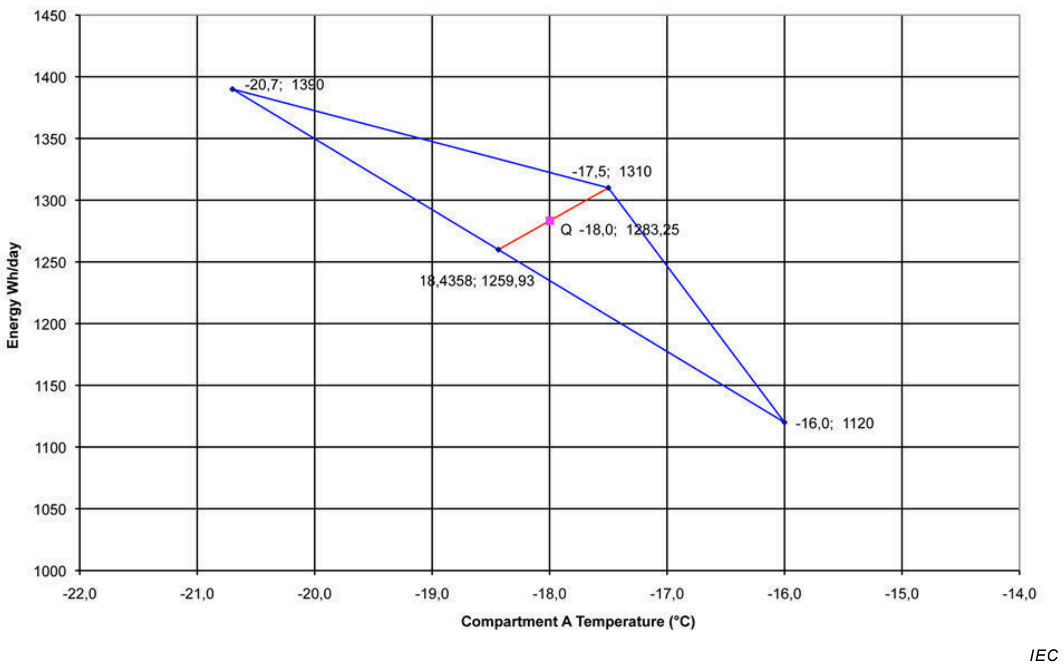

Figure I.9 – Example of triangulation (temperature and energy)

The interpolated **energy consumption** at the temperature for Point 4 between test Points 1 and 3 is determined as follows (**compartment** A temperatures are used):

$$E_4 = E_1 + (E_3 - E_1) \times \frac{(T_{A4} - T_{A1})}{(T_{A3} - T_{A1})} \tag{35}$$

$$E_4 = 1390 + (1120 - 1390) \times \frac{((-18{,}4358) - (-20{,}7))}{((-16{,}0) - (-20{,}7))} = 1\ 259{,}93 \text{ Wh/d}$$

The calculated **energy consumption** at the **target temperature** (Point Q) using temperature and energy data for Point 4 above and test Point 2 is determined as follows (**compartment** A temperatures are used) is given by:

$$E_{AB-tar} = E_2 + (E_4 - E_2) \times \frac{(T_{A-tar} - T_{A2})}{(T_{A4} - T_{A2})} \tag{36}$$

$$E_{AB-tar} = 1310 + (1259{,}93 - 1310) \times \frac{((-18{,}0) - (-17{,}5))}{((-18{,}4358) - (-17{,}5))} = 1\ 283{,}25 \text{ Wh/d}$$

$E_{AB\text{-}tar}$ is the **energy consumption** determined using triangulation of **compartments** A and B. This is illustrated in Figure I.9. Note that the results above for T_{A4} , E_4 and $E_{AB\text{-}tar}$ are normally calculated without rounding. Small differences will occur if the rounded values shown above are used in the equations in this standard. Unrounded values should be used for all calculations where possible. Calculations are normally undertaken in a spreadsheet or other mathematical tool.

I.3.4 Two compartments – triangulation using matrices

For this worked example, we consider the same **refrigerator-freezer** with two **compartments** used for triangulation in the previous example. The use of Formula (33) has already confirmed that the 3 test points surround Point Q. Note that it is not necessary to calculate a value for Point 4 when matrices are used.

The basic premise of the approach on two **compartments** using matrices is to assume that we have 3 simultaneous equations to describe the 3 test points as follows:

$$E_0 + A \times T_{A1} + B \times T_{B1} = E_1$$

$$E_0 + A \times T_{A2} + B \times T_{B2} = E_2$$

$$E_0 + A \times T_{A3} + B \times T_{B3} = E_3$$

In this example, the equations are:

$$E_0 + A \times (-20{,}7) + B \times 6{,}5 = 1\ 390$$

$$E_0 + A \times (-17{,}5) + B \times 0{,}8 = 1\ 310$$

$$E_0 + A \times (-16{,}0) + B \times 7{,}1 = 1\ 120$$

The value of E_0 is conceptually the **energy consumption** of the **refrigerating appliance** at the given ambient test temperature when the temperature of both **compartments** is 0 °C (which will not be possible to achieve in practice).

These three equations can be organised into a matrices as follows:

$$[M_{33}] \times [C_{31}] = [E_{31}] \tag{37}$$

Where:

$[M_{33}]$ is a 3 × 3 matrix of 1 (constant), T_A and T_B for each test point

$[C_{31}]$ is a 3 × 1 matrix of E_0, A and B (constants to be solved)

$[E_{31}]$ is a 3 × 1 matrix of E_1, E_2 and E_3

$$\begin{bmatrix} 1 & -20{,}7 & 6{,}5 \\ 1 & -17{,}5 & 0{,}8 \\ 1 & -16{,}0 & 7{,}1 \end{bmatrix} \times \begin{bmatrix} E_0 \\ A \\ B \end{bmatrix} = \begin{bmatrix} 1390 \\ 1310 \\ 1120 \end{bmatrix}$$

To solve for the unknown constants matrix $[C_{31}]$, find the solution to the matrix multiplication $[M_{33}]^{-1} \times [E_{31}]$

In this example, $[M_{33}]^{-1}$ is equal to:

$$\begin{bmatrix} -3{,}88192 & +1{,}49669 & +3{,}38523 \\ -0{,}21944 & +0{,}02090 & +0{,}19854 \\ +0{,}05225 & -0{,}16371 & +0{,}11146 \end{bmatrix}$$

The matrix multiplication $[M_{33}]^{-1} \times [E_{31}]$ yields the following matrix for E_0, A and B

$$[C_{31}] = \begin{bmatrix} 356{,}2522 \\ -55{,}2769 \\ -16{,}9976 \end{bmatrix}$$

Using the solved constants from matrix $[C_{31}]$, the **energy consumption** at any combination of **compartment** temperatures can be accurately estimated by the equation:

$$E_{AB} = 356{,}2522 - 55{,}276\ 9 \times T_A - 16{,}9976 \times T_B$$

The **energy consumption** at the **target temperature** for **Compartment** A = –18,0 and **Compartment** B = +4,0 is given by:

$$E_{AB\text{-}tar} = 356{,}2522 - 55{,}2769 \times (-18{,}0) - 16{,}9976 \times 4{,}0 = 1\ 283{,}246 \text{ Wh/d}$$

NOTE The result using matrices gives exactly the same result as manual interpolation as set out in the previous subclause. In the examples documented in this subclause and the previous subclause, some errors in the last significant figure may occur due to rounding. This would not occur if spreadsheets are used to calculate the results without rounding.

The energy impact of a change in **compartment** temperatures can be readily calculated from these parameters.

For **Compartment** A (**freezer**), the change in energy resulting from a 1 K warmer **compartment** temperature is given by:

$$\frac{A}{E_{target}} = \frac{-55{,}2769}{1283{,}246} = -4{,}31\ \%$$

i.e. 1 K warmer **freezer** temperature will result in a 4,31 % decrease in **energy consumption** (for a constant fresh food temperature).

Similarly, for **Compartment** B (fresh food), the change in energy resulting from a 1 K warmer **compartment** temperature is given by:

$$\frac{B}{E_{target}} = \frac{-16{,}997}{1283{,}246} = -1{,}32\ \%$$

i.e. 1 K warmer fresh food temperature will result in a 1,32 % decrease in **energy consumption** (for a constant **freezer** temperature).

I.3.5 Three compartments – triangulation using matrices

For this worked example, we consider a **refrigerator-freezer** with three **compartments** and four points used for triangulation, as shown in Table I.8.

Table I.8 – Example of triangulation, three compartments

Parameter	Test 1	Test 2	Test 3	Test 4	Type	Target
Compartment A	–20,1	–18,8	–16,0	–17,4	**Freezer**	–18,0
Compartment B	+4,3	+1,3	+6,4	+2,4	**Fresh Food**	+4,0
Compartment C	–14,2	–12,5	–10,5	–10,5	**Two-star**	–12,0
Energy Wh/d	1 250	1 220	1 080	1 150		

Firstly we check that the Point Q lies inside the tetrahedron formed by the four test points. Calculate the Determinant of the following five matrices:

$$D_0 \text{ for } \begin{vmatrix} -20{,}1 & 4{,}3 & -14{,}2 & 1 \\ -18{,}8 & 1{,}3 & -12{,}5 & 1 \\ -16{,}0 & 6{,}4 & -10{,}5 & 1 \\ -17{,}4 & 2{,}4 & -10{,}5 & 1 \end{vmatrix} = -11{,}898$$

$$D_1 \text{ for } \begin{vmatrix} -18{,}0 & 4{,}0 & -12{,}0 & 1 \\ -18{,}8 & 1{,}3 & -12{,}5 & 1 \\ -16{,}0 & 6{,}4 & -10{,}5 & 1 \\ -17{,}4 & 2{,}4 & -10{,}5 & 1 \end{vmatrix} = -3{,}190$$

$$D_2 \text{ for } \begin{vmatrix} -20{,}1 & 4{,}3 & -14{,}2 & 1 \\ -18{,}0 & 4{,}0 & -12{,}0 & 1 \\ -16{,}0 & 6{,}4 & -10{,}5 & 1 \\ -17{,}4 & 2{,}4 & -10{,}5 & 1 \end{vmatrix} = -3{,}022$$

$$D_3 \text{ for } \begin{vmatrix} -20{,}1 & 4{,}3 & -14{,}2 & 1 \\ -18{,}8 & 1{,}3 & -12{,}5 & 1 \\ -18{,}0 & 4{,}0 & -12{,}0 & 1 \\ -17{,}4 & 2{,}4 & -10{,}5 & 1 \end{vmatrix} = -4{,}075$$

$$D_4 \text{ for } \begin{vmatrix} -20{,}1 & 4{,}3 & -14{,}2 & 1 \\ -18{,}8 & 1{,}3 & -12{,}5 & 1 \\ -16{,}0 & 6{,}4 & -10{,}5 & 1 \\ -18{,}0 & 4{,}0 & -12{,}0 & 1 \end{vmatrix} = -1{,}611$$

As a check $D_0 = D_1 + D_2 + D_3 + D_4$

–11,898 = –3,190 – 3,022 – 4,075 – 1,611 = correct

If D_1 and D_2 and D_3 and D_4 are the same sign as D_0, then Point Q is inside of the tetrahedron (correct).

As per the previous example, the data can be organised into a matrices as follows:

$$[M_{44}] \times [C_{41}] = [E_{41}] \tag{39}$$

$[M_{44}]$ is a 4 × 4 matrix of 1 (constant), T_A, T_B and T_C for each test point

$[C_{41}]$ is a 4 × 1 matrix of E_0, A, B and C (constants to be solved)

$[E_{41}]$ is a 4 × 1 matrix of E_1, E_2, E_3 and E_4.

$$\begin{bmatrix} -20{,}1 & +4{,}3 & -14{,}2 & 1 \\ -18{,}8 & +1{,}3 & -12{,}5 & 1 \\ -16{,}0 & +6{,}4 & -10{,}5 & 1 \\ -17{,}4 & +2{,}4 & -10{,}5 & 1 \end{bmatrix} \times \begin{bmatrix} E_0 \\ A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1\,250 \\ 1\,220 \\ 1\,080 \\ 1\,150 \end{bmatrix}$$

To solve for the unknown constants matrix $[C_{41}]$, find the solution to the matrix multiplication $[M_{44}]^{-1} \times [E_{41}]$

In this example, $[M_{44}]^{-1}$ is equal to:

$$\begin{bmatrix} -8{,}68129 & +10{,}81039 & +6{,}49647 & -7{,}62557 \\ -0{,}67238 & +1{,}24391 & +0{,}66146 & -1{,}23298 \\ +0{,}23533 & -0{,}43537 & +0{,}01849 & +0{,}18154 \\ +0{,}34123 & -1{,}13128 & -0{,}47319 & +1{,}26324 \end{bmatrix}$$

The matrix multiplication $[M_{44}]^{-1} \times [E_{41}]$ yields the following matrix for E_0, A, B and C

$$[C_{41}] = \begin{bmatrix} 583{,}8452 \\ -26{,}4666 \\ -8{,}23668 \\ -11{,}9432 \end{bmatrix}$$

Using the solved constants from matrix $[C_{41}]$, the **energy consumption** at any combination of **compartment** temperatures can be accurately estimated by the equation:

$$E_{ABC} = 583{,}8452 - 26{,}4666 \times T_A - 8{,}23668 \times T_B - 11{,}9432 \times T_C$$

The **energy consumption** at the **target temperature** for **Compartment** A = –18,0 and **Compartment** B = +4,0 and **Compartment** C = –12,0 is given by:

$$E_{ABC\text{-}tar} = = 583{,}8452 - 26{,}4666 \times (-18) - 8{,}23668 \times (+4) - 11{,}9432 \times (-12) \text{ Wh/d}$$

= 1 170,616 Wh/d

The energy impact of a change in **compartment** temperatures can be readily calculated from these parameters.

For **compartment** A, a 1 K warmer **compartment** temperature will result in a 26,4666 Wh/d decrease in energy consumption (equivalent to 1,10 W decrease or a 2,26 % energy decrease per K warmer).

For **compartment** B, a 1 K warmer **compartment** temperature will result in a 8,236 68 Wh/d decrease in energy consumption (equivalent to 0,343 W decrease or a 0,70 % energy decrease per K warmer).

For **compartment** C, a 1 K warmer **compartment** temperature will result in a 11,9432 Wh/d decrease in energy consumption (equivalent to 0,498 W decrease or a 1,02 % energy decrease per K warmer).

I.4 Calculating the energy impact of internal temperature changes

I.4.1 General

It is often useful to calculate the energy impact of internal **compartment** temperature changes which result from changes in user adjustments to **temperature control settings**. Calculation of these values can give a good indication of the user-related impact of changes in **temperature control settings** that may occur from user to user and can assist with analysis of field data.

Analysis of a range of **refrigerator-freezers** tested at an ambient of 32 °C showed that the impact of **freezer** temperature was typically an increase in energy of 2 % to 5 % per degree K **compartment** decrease and for the fresh food temperature was typically an increase in energy of 1 % to 3 % per degree K decrease. These values vary by model.

While such calculations are of interest and are recommended, they are not required as part of this standard.

NOTE When calculating the energy impact of internal temperature changes, great care is required in cases where the base of the triangle is less than 2 K and the height of the triangle is less than 1 K. Small or flat shaped triangles may not provide an accurate estimate of the impact in either **compartment** for products with 2 **user-adjustable temperature controls**.

I.4.2 One compartment

Where two point interpolation using a single control is used to calculate the energy for a **refrigerating appliance** with only one **compartment**, the energy impact per degree K change can be readily calculated.

$$E_{target} = E_1 + (E_2 - E_1) \times \frac{(T_{tar} - T_1)}{(T_2 - T_1)}$$

and

$$\Delta E = \frac{(E_2 - E_1)}{(T_2 - T_1) \times E_{target}}$$

where

E_{target} is the **energy consumption** at the **target temperature** determined by linear interpolation from test Points 1 & 2

E_1 is the measured **energy consumption** at test Point 1 for **temperature control setting** 1

E_2 is the measured **energy consumption** at test Point 2 for **temperature control setting** 2